# Capital gains taxes and the cost of delegated portfolio selection*

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# Capital gains taxes and the cost of delegated portfolio selection


**Abstract**

We consider the portfolio choice of an individual who can invest in two risky assets (in addition to a riskless asset) and who is subject to taxes on realized capital gains. These taxes appear in the portfolio choice problem as a form of state-dependent, endogenous transaction costs. Similar to the case of portfolio choice with transaction costs, the optimal strategy of the taxable investor contains a "no trade" region originating from the exercise of the option to defer capital gains taxes. This may lead an investor to hold a markedly undiversified portfolio for reasonable parameter values. With multiple risky assets the investor is effectively holding a *portfolio* of tax-deferral options. We show that the main factors affecting optimal choices in the presence of taxes are: (i) the need to sell asset to finance consumption, (ii) risk aversion, (iii) the investment horizon and (iv) the variance-covariance structure of asset returns. The value of the tax-deferral options is decreasing in time to maturity, volatility and correlation of the assets and in the risk aversion. We also document that the loss in flexibility due to investing in a non tax-efficient mutual fund increase considerably (up to 10% of initial wealth) as the correlation between the managed assets decreases.


*Keywords*: Portfolio choice, capital gains taxes, investment management.
*JEL Classification*: G11

# 1 Introduction

Individual investors are subject to a non-trivial amount of taxes whenever they sell securities they hold at a profit. In the US, the rate of (long-term) capital gains taxes currently stands at 15% (5% for individuals in the 15% bracket). At these rates, capital gains taxes represent an important cost for investors in financial markets. In order to obtain an optimally diversified portfolio at minimum cost, investors must consider the tax status of each asset in their portfolio, as well as their interaction.

The most important feature of how capital gains taxes are levied in practice is that they are payable only upon realization and not on accrual. The optimal portfolio strategy hence involves dynamic exercise of a portfolio of tax-timing options. Constantinides (1983) shows that, if investors are allowed to short-sell their assets and have full use of the proceeds from the short-sale, the optimal portfolio investment policy can be separated from the optimal liquidation policy. In this case, the optimal tax-realization policy is to defer gains indefinitely and immediately realize losses.[1] In the presence of short-sell restrictions, however, the portfolio problem and the exercise of timing options is no longer separable. Investors will therefore face a trade-off between optimal exercise of tax-timing options and optimal diversification: investors who exercise their tax-timing option by delaying realization of gains and immediately realizing losses may end up holding highly undiversified portfolios.

A striking reality of modern financial markets is the fact that a large proportion of household wealth is invested through the help of financial intermediaries (e.g. mutual funds). According to the U.S. tax code, to avoid corporate taxation, mutual funds are required to distribute a minimum of 90% of their income and capital gains to shareholders. Therefore, an investor who invests in a fund is exposed to potentially sub-optimal tax realization policies imposed by the trading activity of the fund manager. In other words, the investor in a mutual fund is forced to realize gains on trades that have been delegated to the manager.

The purpose of this paper is to assess the potential tax-inefficiency of delegated portfolio investment. To this purpose, we first follow the existing literature (see for example Dammon,

[1] Before the 1997 Tax Reform Act, investors could engage in "shorting against the box" strategies, virtually eliminating the incidence of capital gains taxes. See Gallmeyer, Kaniel, and Tompaidis (2004) for an analysis of the cost of short-selling on optimal portfolios with capital gains taxes.

Spatt, and Zhang (2001, 2002), Gallmeyer, Kaniel, and Tompaidis (2004)) to solve for the optimal portfolio with two risky assets and capital gains taxes.[2] We then use the solution to this problem as a benchmark for analyzing the potential cost incurred by an investor who delegates his portfolio selection problem to a tax-exempt mutual fund. We conduct our analysis by assuming different potential trading strategies followed by the mutual fund manager (). We also assess the tax-efficiency that can be achieved through the use of more recent investment vehicles such as exchange traded funds (ETFs) who invest in broad market indices.

The optimal portfolio is characterized by the existence of a "no-trade" region originating from the exercise of the option to defer capital gains taxes. The no-trade region is itself state dependent and reflects the interdependence of tax status and holdings of the assets. A large no-trade region means that investors are willing to hold an undiversified portfolio in order to minimize tax. We show that the lack of diversification resulting from the exercise of the tax-deferral option can be significant. For reasonable parameter values, we compute that this diversification cost is in the order of five to ten percent of the initial wealth. The value is affected by the need for intermediate consumption, risk aversion, and the investment horizon.

With multiple risky assets, the investor has additional flexibility in achieving optimal risk exposure at minimal tax cost. When assets are negatively correlated, they act as complements; instead of selling an asset with an embedded gain, optimal risk exposure can be achieved by buying a negatively correlated asset. When assets are positively correlated, instead of selling an asset with an embedded gain, a substitute asset with a lower embedded gain can be sold. This qualifies as a form of imperfect shorting against the box.[3] The change in strategies triggered by cross-asset effects results in a change in diversification costs. When the correlation between assets is high, the tax-deferral options embedded in the two securities tend to be in the money or out of the money at the same time leading to a higher diversification cost of exercising the options. On the other side, when assets are negatively correlated, the investor can fully exploit the benefit of optimally and independently exercising the tax deferral option on the underlying securities.

[2] In solving the portfolio problem we model the cost basis as a weighted average of the purchase prices of the two assets and assume that investors are allowed to perform wash-sales. As DeMiguel and Uppal (2005) show, the difference in strategies and wealth between using an average tax basis and the exact tax basis is negligible.

[3] This confirms the results of Gallmeyer, Kaniel, and Tompaidis (2004).

To assess the tax-cost of delegated investing we compare the optimal strategy of an investor who can trade in two risky assets with the same investor "forced" to trade through a mutual fund. We assess the potential inefficiency of a mutual fund under a variety of alternative strategies () as well as different statistical properties of asset returns. We find that ....

The papers that are most closely related to ours are Dammon, Spatt, and Zhang (2001, 2002) and Gallmeyer, Kaniel, and Tompaidis (2004). Dammon, Spatt, and Zhang (2001) analyze the optimal dynamic consumption, investment and liquidation policies for the case of a single risky asset while Dammon, Spatt, and Zhang (2002) extend their original set-up to a two-asset problem. Consistent with our analysis in the case of an investor with intermediate consumption, they show that diversification benefits can significantly outweigh the tax cost of selling. Rather than focusing on a lifetime portfolio consumption problem, we focus on the trade-off between diversification costs, tax realization costs, and the cross-effects of risky asset holdings with specific emphasis to the role of delegated investing. Gallmeyer, Kaniel, and Tompaidis (2004) also consider the two risky assets and study the effect of relaxing the short selling constraint. Our analysis of the tax cost of delegated investing is motivated by Leland (2000) who examines the optimal implementation strategy of a mutual fund manager who wishes to maintain assets in exogenous fixed proportions. In our approach we analyze a wide variety of trading strategies implemented by a mutual fund and assess their tax consequences. Finally, since our focus is on the trade-off between optimal diversification and optimal tax-realization in a portfolio problem, we abstract from asymmetric taxation of short- and long-term gains and from the existence of tax-deferred accounts.[4]

The rest of the paper proceeds as follows. In Section 2 we develop the model for the case of two risky assets and a riskless asset. In Section 3 we numerically solve the model and analyze the properties of the optimal trading strategy and the value of tax-deferral option. In addition, we describe the portfolio problem of an agent that can invest in an

[4]There is an extensive body of earlier literature that deals with the portfolio allocation problem in the presence of capital gains taxes. Constantinides (1983, 1984) pioneered the study of optimal investment and liquidation policy in the presence of capital gains taxes. Dammon, Dunn, and Spatt (1989) quantitatively assess the value of the tax-timing option by explicitly considering the different tax treatment of long and short-term capital gains. Dammon and Spatt (1996) investigate theoretically the optimal trading and pricing of securities subject to asymmetric taxation in the context of a no-arbitrage model while Dammon, Spatt, and Zhang (2004), Huang (2003) and Garlappi and Huang (2006) analyze a single-risky asset portfolio *location* problem in which the investor has to decide how to invest in tax-deferred and taxable accounts.

open-ended mutual fund and assess the flexibility costs induced by the presence of capital gains taxes. Section 5 concludes. In the appendixes, we discuss the key properties of the set of feasible strategies, reformulate the portfolio problem as a dynamic programming problem, and analyze the effect of labor income on the portfolio problem.

## 2 The Model

We consider the optimal portfolio choice problem of a risk averse investor with investment horizon of $T > 0$ periods. The agent derives utility only from consumption at the terminal period $T$ and can trade in three kinds of assets available in the financial markets: two risky stocks (labeled "Asset 1" and "Asset 2") and a riskless money market instrument (labeled "Asset 0"). The agent is subject to an income tax on dividends and a capital gains tax (rebate) on realized gains (losses). In addition, the agent can borrow but is subject to short-sale constraints on the risky assets. The set-up for our analysis is identical to Dammon, Spatt, and Zhang (2001) and Gallmeyer, Kaniel, and Tompaidis (2004).

### 2.1 Distribution of asset returns

We indicate with $S_0(t)$ the pre-tax price of the money market instrument at time $t$. Its evolution over time is given by

$$S_0(t+1) = S_0(t)e^r, \quad t = 0, \ldots, T-1 \tag{1}$$

with $r$ denoting the (constant) pre-tax risk-free rate per period. At time $t$, the tow risky assets have prices $S_i(t)$, $i = 1, 2$ and payout a pre-tax dividend equal to $\delta_i S_i(t-1)$. The evolution of the pre-tax, ex-dividend prices of the risky assets is given by

$$S_i(t+1) = S_i(t)\exp\left\{\mu_i - \frac{1}{2}\sigma_i^2 + \tilde{\epsilon}_i\right\}, \quad t = 0, \ldots, T-1 \tag{2}$$

$$(\epsilon_1, \epsilon_2)^\top \sim \mathcal{N}(\mathbf{0}_{2\times 1}, \Sigma) \tag{3}$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{4}$$

with $\mu_i$ indicating the capital gain return per period, $\sigma_i$ the volatility of returns, $\rho$ the correlations and $\mathcal{N}(\cdot)$ the multivariate normal probability distribution. Dividend and the interest on the money market instrument are subject to the same income-tax rate $\tau_d$ while capital gains are subject to a tax rate $\tau_g \neq \tau_d$.[5]

## 2.2 Basis evolution and capital gains taxes

In order to calculate the capital gains/losses on an asset, it is necessary to define its "cost basis". The basis at every time is represented by the dollar cost of stocks held at time $t$ (i.e., the *Book Value of shares*). We indicate by $B_i(t)$ the basis for asset $i$ at time $t$. If an asset is sold at time $t$ and has an embedded gain ($S_i(t) > B_i(t)$) then the investor incurs in capital gains taxes at a rate $\tau_g$. On the other side if an asset has an embedded loss (($S_i(t) < B_i(t)$)), it is optimal for the investor to immediately realize such loss by selling the asset and immediately re-purchasing it.[6] This will reset the basis to the current price of the asset. Capital gains/losses are computed on the difference between the price of the asset and the corresponding basis.

Ideally, as it is allowed by the tax code, the investor would like to track the costs of each group of purchased shares, and then choose what to sell according to the embedded gains/losses (specific identification criterion). However, this problem is path-dependent and it quickly becomes intractable to calculate optimal strategies. Instead, we assume the investor elects to use the average cost method for calculating cost basis, and we model the basis as a weighted average of purchase prices (average criterion).[7] Using this simplification, and denoting by $n_i(t)$ the number of shares of asset $i$ held in the portfolio at time $t$ we can derive the following evolution of the tax basis:

$$B_i(t) = \begin{cases} \frac{n_i(t-1)B_i(t-1)+[n(t)-n(t-1)]^+ S_i(t)}{n_i(t-1)+[n(t)-n(t-1)]^+}, & \text{if } B_i(t-1) < S_i(t) \\ S_i(t), & \text{if } B_i(t-1) > S_i(t) \end{cases}, \tag{5}$$

where $[\cdot]^+ = \max[\cdot, 0]$. The upper equation in (5) computes the average basis when there is an embedded gain. When the investor purchases asset $i$, $[n(t) - n(t-1)]^+ > 0$ and the new bases is simply equal to the average cost of existing shares. If the investor sells, $[n(t) - n(t-1)]^+ = 0$ and the basis remains unaffected since it is implicitly assumed that

[5].

[6]We are allow for wash-sales for assets with embedded capital losses, i.e., the investor is allowed to sell and immediately repurchase an asset for the purpose of realizing capital losses and resetting the basis.

[7]As DeMiguel and Uppal (2005) show, the optimal portfolio strategies for an investor with CRRA preferences are hardly affected by the method used in determine the cost basis.

the cost of shares sold is equal to their average price. The second line of equation (5) shows that the basis is reset by wash sales when asset $i$ has an embedded loss.

We denote by $G_i(t)$ the capital gain/loss on risky asset $i = 1, 2$ arising after rebalancing at time $t$. Capital gains at time $t$ $G(t)$ are computed as follows

$$\begin{aligned} G(t) \quad &= \quad \sum_{i=1}^{2}(S_i(t) - B_i(t-1)) \cdot \mathbf{1}_{\{B_i(t-1)<S_i(t)\}} \cdot [n(t-1) - n(t)]^{+} + \\ &\quad \sum_{i=1}^{2}(S_i(t) - B_i(t-1)) \cdot \mathbf{1}_{\{B_i(t-1)>S_i(t)\}} \cdot n(t-1) \end{aligned} \tag{6}$$

The first part refers to capital gains realization and the second to tax-loss realizations (wash sales) and captures the tax subsidy obtained by realizing losses.[8]

## 2.3 Wealth evolution

Let $W(t)$ denote the beginning of period wealth, before capital gain taxes are paid, after dividend taxes and before consumption/ portfolio rebalancing decisions. Since the strategies are self-financing the wealth after rebalancing has to equal the wealth before rebalancing:

$$W(t) = n_0(t)S_0(t) + \sum_{i=1}^{2} n_i(t)S_i(t) + \tau_g G(t) + C(t), \tag{7}$$

where $C(t)$ is the consumption in period $t$. The wealth $W(t+1)$ at the beginning of next period (before rebalancing and after dividend taxes) is

$$\begin{aligned} W(t+1) \quad &= \quad n_0(t)S_0(t)\left(e^{r}(1-\tau_d) + \tau_d\right) \\ &\quad \sum_{i=1}^{2} n_i(t)\left(S_i(t+1) + S_i(t)\delta_i(1-\tau_d)\right) \end{aligned} \tag{8}$$

Solving for $n_0(t)S_0(t)$ in (7) and substituting this into (8) we obtain the following expression for the evolution of wealth

$$\begin{aligned} W(t+1) \quad &= \quad \left(W(t) - \sum_{i=1}^{2} n_i(t)S_i(t) - \tau_g G(t) - C(t)\right)\left(e^{r}(1-\tau_d) + \tau_d\right) + \\ &\quad \sum_{i=1}^{2} n_i(t)\left(S_i(t+1) + S_i(t)\delta_i(1-\tau_d)\right). \end{aligned} \tag{9}$$

[8]In reality, the tax subsidy will only be received by the investor as an offset against corresponding gains in the current period, or against gains carried forward or carried back from other periods. We ignore the details of the tax code in this case.

## 2.4 Preferences and individual optimization

To understand the effect of capital gains taxes on different kinds of investors, we consider two scenarios. In the first scenario, the investor maximizes the expected utility of the consumption stream he can obtain from his financial assets. In the second scenario, the investor maximizes the expected utility of terminal wealth and has no intermediate consumption. The first case represents the portfolio choice problem of an investor in the later part of is life-cycle and the second case proxies for the problem of an investor in the early to middle part of his life cycle where he is in a wealth-building stage, and has access to his human capital (i.e. labor income) to satisfy his consumption demand.[9]

The investor chooses a strategy $(C(t), n_0(t), n_1(t), n_2(t))$, $t = 0, \ldots, T-1$, with $C(t) \geq 0$, $n_i(t) \geq 0$ for $i = 1, 2$ (no short-sales allowed on the risky assets) that maximizes his intertemporal utility

$$\max_{\{C(t), n_1(t), n_2(t)\}_{t=0}^{T-1}} E\left[\sum_{t=0}^{T-1} \beta^t \frac{C(t)^{1-\gamma}}{1-\gamma} \cdot \mathbf{1}_{\{\text{cons}\}} + \beta^T \frac{W(T)^{1-\gamma}}{1-\gamma}\right], \tag{10}$$

subject to (9), where $\mathbf{1}_{\{\text{cons}\}}$ is an indicator that we set to one to analyze the problem with intermediate consumption and to zero to study the case in which the investor maximizes terminal wealth.

## 2.5 Numerical implementation

The problem in (10) does not admit a closed form solution and we need to resort to numerical methods. We can use this to reduce the dimensionality of the problem by noting that the problem is homogeneous in $W(t)$. Let us define the following three quantities:

$$x_i(t) = \frac{n_i(t-1)S(t)}{W(t)}, \quad y_i(t) = \frac{n_i(t)S(t)}{W(t)}, \text{ and } \alpha_i(t) = y_i(t) - x_i(t). \tag{11}$$

The quantity $x_i(t)$ represents the portfolio holding in asset $i$ *before rebalancing* at time $t$ (a state variable), $y_i(t)$ represents the portfolio holding in asset $i$ *after rebalancing* at time $t$

[9] The solution of the portfolio problem with multiple risky assets and non-financial income would involve five state variables, making the problem numerically intractable, given our computational capacity. In an earlier version of this paper we solved for the consumption-portfolio problem with one risky asset and non-financial (e.g. labor) income and showed that the lock-in effect generated by capital gains taxes is stronger in the absence of non-financial income. This gives us confidence in assuming that the two-risky asset case with no intermediate consumption is an acceptable proxy for the case of a consumption-investment problem with two risky assets *and* non-financial income.

(a control variable).[10] and $\alpha_i(t)$ represents the *trade* (as a percentage of $W(t)$) in asset $i$ at time $t$.

To improve the stability of the numerical solution it is convenient to normalize the basis $B_i(t)$ by the price $S_i(t+1)$ and work wth the basis-to-price ratio

$$\theta_i(t+1) = \frac{B_i(t)}{S_i(t+1)}. \tag{12}$$

Using this definition and the normalization in (11) we can derive the evolution of $\theta_i(t)$ as follows

$$\theta_i(t+1) = \begin{cases} \frac{x_i(t)\theta_i(t)+[y_i(t)-x_i(t)]^+}{\frac{S_i(t+1)}{S_i(t)}(x_i(t)+[y_i(t)-x_i(t)]^+)}, & \text{if } \theta_i(t) < 1 \\ \frac{S_i(t)}{S_i(t+1)}, & \text{if } \theta_i(t) > 1 \end{cases} \tag{13}$$

where $\frac{S_i(t+1)}{S_i(t)}$ is the ex-dividend return defined in (2). Similarly, the expression for capital gains $G(t)$ in (6) can be re-written as

$$\begin{aligned} g(t) \equiv \frac{G(t)}{W(t)} &= \sum_{i=1}^{2}(1-\theta_i(t)) \cdot \mathbf{1}_{\{\theta_i(t)<1\}}[x_i(t)-y_i(t)]^+ + \\ & \sum_{i=1}^{2}(1-\theta_i(t)) \cdot \mathbf{1}_{\{\theta_i(t)>1\}}x_i(t). \end{aligned} \tag{14}$$

Defining $c(t) = \frac{C(t)}{W(t)}$, the wealth evolution in (9) can be re-written as follows

$$\begin{aligned} R_W(t+1) \equiv \frac{W(t+1)}{W(t)} &= \frac{1}{1-\sum_{i=1}^{2}x_i(t+1)} \cdot \\ & \left[\left(1-\sum_{i=1}^{2}y_i(t)-\tau_G g(t)-c(t)\right)(e^r(1-\tau_D)+\tau_D)+ \right. \\ & \left. \sum_{i=1}^{2}y_i(t)\delta_i(1-\tau_D)\right] \end{aligned} \tag{15}$$

where,

$$x_i(t+1) = \frac{n_i(t)S_i(t+1)}{W(t+1)} = \frac{y_i(t)\frac{S_i(t+1)}{S_i(t)}}{\frac{W(t+1)}{W(t)}} \tag{16}$$

[10] Note that we express these weights as a fraction of $W(t)$ and not of the un-consumed wealth, as it is common in traditional portfolio problems. If we define by $w$ the traditional portfolio weights (after rebalancing) as a fraction of unconsumed wealth then $w = \frac{y}{1-c}$. This will be relevant when we look at the Merton solution, for example.

Substituting (16) in (15) and simplifying we obtain

$$R_W(t+1) \equiv \frac{W(t+1)}{W(t)} = \left(1 - \sum_{i=1}^{2} y_i(t) - \tau_G g(t) - c(t)\right)(e^r(1-\tau_D)+\tau_D) + \sum_{i=1}^{2} y_i(t)\left[\frac{S_i(t+1)}{S_i(t)} + \delta_i(1-\tau_D)\right] \tag{17}$$

where, again, $\frac{S_i(t+1)}{S_i(t)}$ is the ex-dividend return defined in (2).

## 2.6 Final formulation of the problem

After the normalization and simplifications in the previous section, the portfolio problem with capital gains taxes is characterized by (i) Four state variables: $\mathbf{z} \equiv (x_1, x_2, \theta_1, \theta_2)$ and (ii) Three control variables $c, y_1, y_2$. The problem is characterized by the following Bellman equation

$$v(\mathbf{z}(t), t) = \max_{c(t), y_1(t), y_2(t)} \left\{ \frac{1}{1-\gamma} c(t)^{1-\gamma} \mathbf{1}_{\{\text{cons}\}} + \beta E_t \left[ R_W(t+1)^{1-\gamma} v(\mathbf{z}(t+1), t+1) \right] \right\}, \tag{18}$$

with boundary condition

$$v(\mathbf{z}(T), T) = \frac{1}{1-\gamma}, \tag{19}$$

where

$$x_i(t+1) = y_i(t) \frac{S_i(t+1)}{S_i(t)} \frac{1}{R_W(t+1)} \tag{20}$$

$$R_W(t+1) = \text{Equation (17)} \tag{21}$$

$$\frac{S_i(t+1)}{S_i(t)} = \text{Equation (2)} \tag{22}$$

$$\theta_i(t+1) = \text{Equation (13)} \tag{23}$$

$$g(t) = \text{Equation (14)} \tag{24}$$

and subject to the constraint $W(t) \geq 0$ for all $t$, which is equivalent to the non-bankruptcy requirement

$$R_W(t+1) \geq 0, \text{ in all states and at all dates } t = 0, \ldots, T-1. \tag{25}$$

We solve the above dynamic programming problem numerically by backward induction, and approximate the normal shocks in the returns via a quadrature scheme (). We discretize the state space $\mathbf{z} = [x_1, x_2, \theta_1, \theta_2]$ into a four-dimensional grid, solve the problem at period $T$ and then solve backwards recursively.

We calibrate our benchmark numerical solution by choosing parameters in the range suggested by existing literature. We assume that both risky assets have the same gross return, $\mu_1 = \mu_2 = 10\%$ and the same volatility $\sigma_1 = \sigma_2 = 30\%$. The money market risk-free instrument is assumed to yield a 0% interest rate.[11] The correlation coefficient is set to $\rho = 0.5$. The risk aversion parameter for the investor is $\gamma = 3$. The tax rate on realized gains and losses is assumed to be $\tau_g = 25\%$. We solve a twenty-period model, so $T = 20$. The unit of time is one year.

## 3 Optimal portfolios and value of the tax timing option

In this section we analyze the properties of the optimal portfolio in the presence of capital gains taxes. Our focus will be in identifying the trade-off between optimal diversification cost and optimal tax realizations. Capital gains taxes act as a form of state dependent transaction costs. In the presence of an embedded gain, any sale of the asset triggers a tax payment. An investor who might otherwise choose to sell to obtain an optimally diversified portfolio, may choose to refrain from selling in order to avoid realization of taxable gains. The optimizing investor must therefore weigh the cost of sub-optimal diversification against the cost of immediate tax payments in determining his investment strategy.

### 3.1 One asset

We start our analysis by considering the diversification cost of capital gains taxes in the case in which only one asset is present. This is useful to highlight the fundamental trade-off faced by the investor.

[11] Given our assumption on the distribution of the risky asset return, this amounts to assume an expected risk premium of 10% p.a.

### 3.1.1 Portfolio properties

The basic forces at play in the presence of capital gains taxes can be grasped by looking at Figure 1 and 2.

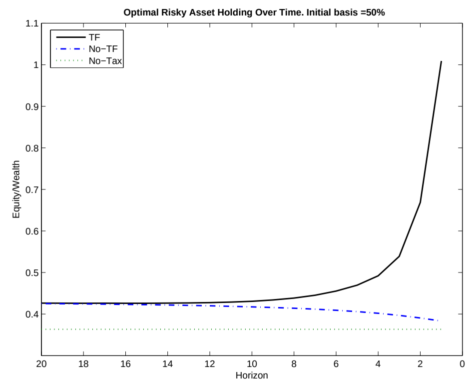



**Figure 1: Single Asset. Portfolio holdings over time.**
This Figure shows the percentage of risky asset held by an investor with different investment horizon. The basis at each trading date is set to 50% of the stock price. The solid line ('TF') represents the case of tax-forgiveness at the end of the horizon. The dash-dotted line ('No-TF') is the case of no tax forgiveness and the dotted line ('No-Tax') is the case of no capital gains taxes.

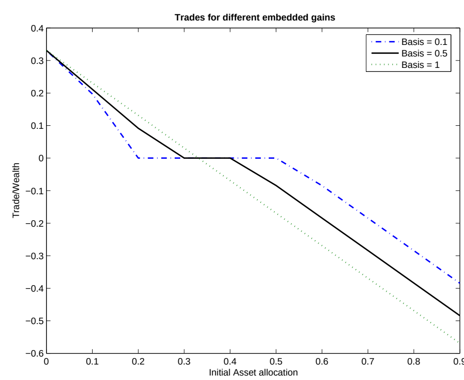



**Figure 2: Single Asset. Trading strategies.**
This Figure shows the trading strategy at time $t = 0$ in the single-asset problem for a variety of different basis to price ratios. Trades are reported as a percentage of liquidating wealth. The horizontal axis reports the initial asset allocation.

The first figure shows the portfolio holding in the risky asset overtime. The figures distinguishes between the case in which there is tax-forgiveness at death (solid line), the case of no tax-forgiveness (dash-dotted line) and the no-tax case (dotted line). It is evident from the figure that tax forgiveness induces investors to hold considerably more stock, the shorter the investment horizon. The picture is drawn assuming that the embedded gain is 50% of the stock price. Obviously, a larger embedded gain will exacerbate the tendency to hold more stocks in the portfolio. It is also interesting to note how the tax-forgiveness provision is crucial for this result. The portfolio holding for the no-tax-forgiveness case, in fact, exhibit a reverse pattern, with lower amount of stock held as the investment horizon shortens. Finally, the case of no taxes shows the well know result of constant asset holding over time (Merton solution).

Figure 2 analyzes the optimal trades at time zero for different level of embedded gains and for different initial risky asset position. The effect of capital gains taxes is to introduce a no-trade region in the optimal strategy of the investor. Such no trade region is represented by the flat part at zero in the lines reported on the graph. Notice that the larger is the embedded gain and the larger the no trade region. The intuition is, of course, that capital gains taxes act as transaction costs, and this cost is higher the higher is the embedded gain, forcing the investor to give up diversification. Finally note that the case of no embedded gain does not show any no-trade region.

#### 3.1.2 Value of tax-timing option

A direct way of examining how diversification motives and tax-timing motives interact in determining the optimal investment strategy is to compare the optimal strategy with the strategy adopted by an investor who pays taxes on accrual, i.e., who is forced to pay taxes each period on capital gains earned in that period and lacks the possibility to defer (taxation *on accrual*).

### 3.2 Two Assets

The opportunity to trade in two asset clearly adds extra flexibility not only in the portfolio selection process but also in the tax realization strategy. We repeat the above analysis for the case of two assets and investigate in detail the incremental benefit to the total deferral option from being able to trade in multiple assets. Our main quantity of interest here will be the value of the tax deferral option compute as a difference in certainty equivalent between two tax regimes: accrual vs. realization. In the next two sections we look at the flexibility of the tax deferral option by comparing the same tax regime (realization) across environment in which the number of assets differ.

### 3.2.1 Optimal portfolio properties

Figure 3 displays the patterns of trades in the two assets as a function of initial holdings $x_1$ and $x_2$. Each panel represents a different combination of tax statuses of the two assets. The black region is the *no-trade* region where the investor chooses not to trade in either risky asset. The grey region shows positions where only one risky asset is traded. In the grey areas on the top and bottom of the no trade region, only asset 2 is sold and bought, respectively, while in the grey areas to the left and right of the no trade region, only asset 1 is bought and sold, respectively. In the white region both assets are traded. The intersection of the horizontal and vertical black lines at $(0.35, 0.35)$ represents the degenerate no-trade region in the accrual case. Since the grey and black regions show conditions where the investor avoids trading for tax reasons, the size of the white region in each Panel gives an idea of the magnitude of diversification loss due to capital gains taxes. From Figure 3 we observe that the lack of diversification due to capital gains taxes can be substantial. In Panel A, where both assets have large embedded gains, the black and grey regions together cover most of the state space. The investor refrains from trading while holding a portfolio containing more that 40% of wealth in each asset. This amounts to more than double the Merton portfolio. Only at extremely high levels of portfolio holdings is the investor willing to pay taxes in order to rebalance the portfolio. When the embedded capital gain is smaller, as in Panel D, the diversification loss is considerably smaller.

Our qualitative results on the trade-off between optimal tax-realization and optimal diversification are robust to changes in our assumptions about intermediate consumption and time horizon. In particular, the main consequence of removing intermediate consumption is that the no-trade regions become larger. Since investors have no need to liquidate assets in order to finance consumption, they will face less frequent tax realization. Similarly, a longer investment horizon will result in smaller no-trade regions. The incentive to defer is stronger the closer the investor is to the time of tax forgiveness.[12]

[12] These results are available upon request.

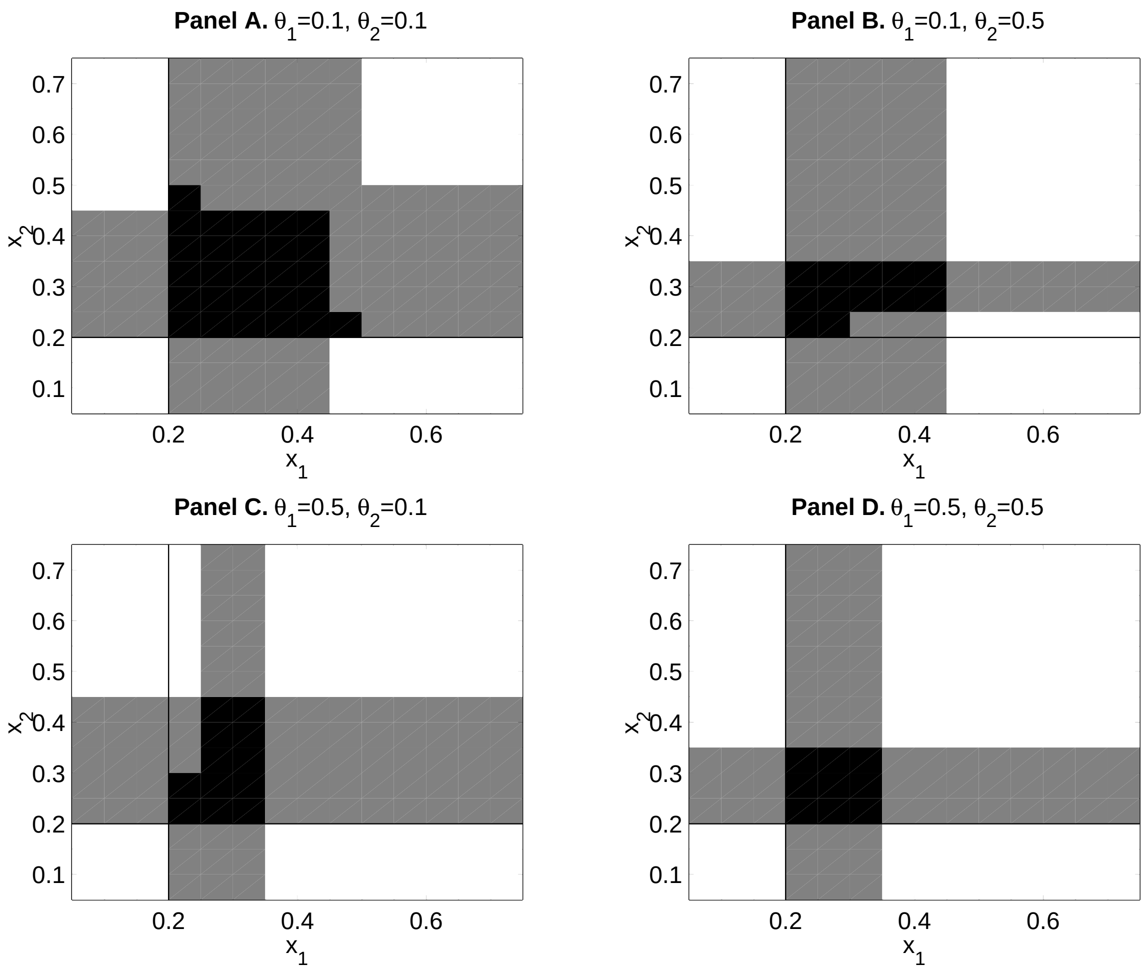
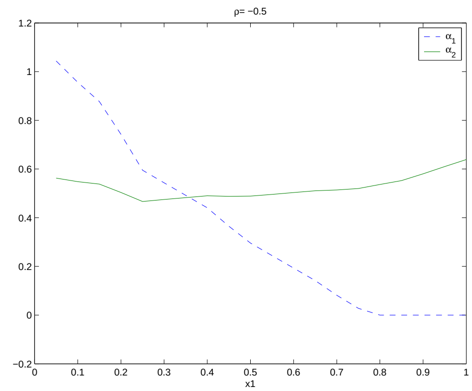



**Figure 3: Regions of No-Trade. Intermediate Consumption case.**
This Figure shows the investment strategy at time $t = 0$ in the two-asset problem for a variety of different basis to price ratios. Three different types of actions are detailed as a function of the initial portfolio positions in each asset. In the black region, the investor chooses not to trade in either asset. In the grey region, one asset is traded (bought or sold), while holdings of the other asset remain constant. The white region shows values of the initial holdings where the investor chooses to trade in both assets. The black cross indicates the investment by the non-taxed (Merton) investor.

Capital gains taxes inhibit the ability of an investor to optimally diversify his portfolio. The presence of multiple assets, however, gives additional flexibility in achieving an optimal risk exposure. This flexibility can only be analyzed in a multi-asset model. These cross-asset effects are evident in the non-convexity of the no-trade region shown in Figure 3.[13] For example, in Panel B, the trading strategy for asset 2 is affected by the holding in asset 1.

[13]This is consistent with the results in Akian, Menaldi, and Sulem (1996) who show a similar property for the case of proportional transaction costs.

Panel A: $\rho = -0.5$




Panel B: $\rho = 0.9$

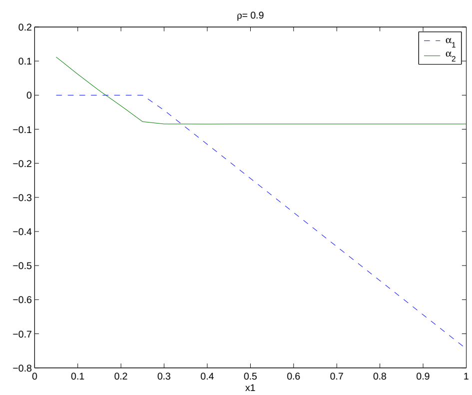



**Figure 4: Cross-effects**

This figure shows the trades in asset 1 ($\alpha_1$) and asset 2 ($\alpha_2$) as a function of the holding of asset 1 ($x_1$). The holdings of asset 2 are fixed at $x_2 = 0.2$, the basis to price ratio for asset 1 is $\theta_1 = 0.2$ and the basis to price ratio for asset 2 is $\theta_2 = 1.0$. The two assets are negatively correlated ($\rho = -0.5$). The other parameter values are as specified in Table 1.

In the following, we examine this effect in more detail by explicitly analyzing, for different values of the correlation among assets, how the investment in one asset is affected by the holding and tax status of the other asset.

In Figure 4 we consider two extreme cases of the relationship between the two assets. The trades in both assets as a function of holdings in asset 1 are shown when (i) asset 1 has an embedded gain ($\theta_1 = .2$), (ii) asset 2 is neutralized ($\theta_2 = 1$), and (iii) the investor is under-exposed in asset 2 ($x_2 = 0.1$), compared to the Merton case. The solid line represents trades in asset 2 ($\alpha_2$) while the dotted line represents trades in asset 1 ($\alpha_1$).Panel A reports the case where assets are negatively correlated ($\rho = -0.5$). In this case, the Merton portfolio is $x_1 = 87.26\%$ and $x_2 = 59.63\%$. The most important feature of this plot is the change in the trade in asset 2 as the holding in asset 1 changes. As the investor becomes more and more exposed to asset 1, he compensates for the over-exposure by purchasing more of asset 2, which acts as a *complement*. Panel B represents the case in which the two assets are strongly positively correlated ($\rho = -0.9$). The Merton portfolio is $x_1 = 7.57\%$ and $x_2 = 20.76\%$. Even though the investor's holding of asset 2 are roughly similar to the Merton portfolio, the investor still sells asset 2. The large embedded gain in asset 1 makes selling that asset too costly. Being negatively correlated and tax-neutral, asset two allows an investor to achieve nearly the same risk exposure at lower cost. This can be thought of as an imperfect form of "shorting against the box" (see also Gallmeyer, Kaniel, and Tompaidis (2004)).

### 3.2.2 Value of tax timing option

As for the case of a single asset we compute the value of the tax-timing option by calculating the percentage of extra wealth necessary to make the investor paying taxes on accrual as well off as the investor who pays taxes on realization. We will refer to this measure as to the *certainty equivalent* benefit of the tax-on realization versus the tax-on-accrual. Values greater than zero indicate states where investing in tax-on-realization assets is preferred. We interpret this quantity as the value of optimal tax-deferral, or, equivalently, the value of the tax-timing option.

In Figure 5, we report the value of the tax-deferral option for different initial states. In each panel, we plot the value as a function of the holding and basis of asset 1, keeping the holding and basis of asset 2 constant. The value of the option to defer is non-negative and is greater when the investor has a high holding of asset 1 and asset 1 has a large embedded gain (north-west corner). We say that in this case the option to defer is *in-the-money*. When the assets have an embedded loss we say that the option to defer is *out-of-the-money*. An option to defer is more valuable when there is something to be deferred, and hence it is intuitive to observe a higher value of the option in the gain region ($\theta_i < 1$) than in the loss region ($\theta_i > 1$).[14] Finally, observe that the utility level is constant in the loss region

[14] This is also a consequence of the fact that we normalize by liquidating wealth. Consider an investor with a capital gain, and an investor with a capital loss. Figure 5 shows that the gain investor is comparatively better off. By construction, both investors have the same liquidating wealth. However, the liquidating wealth for the investor with the gain includes a large tax liability that can be deferred. This option to defer makes the gain investor better off.

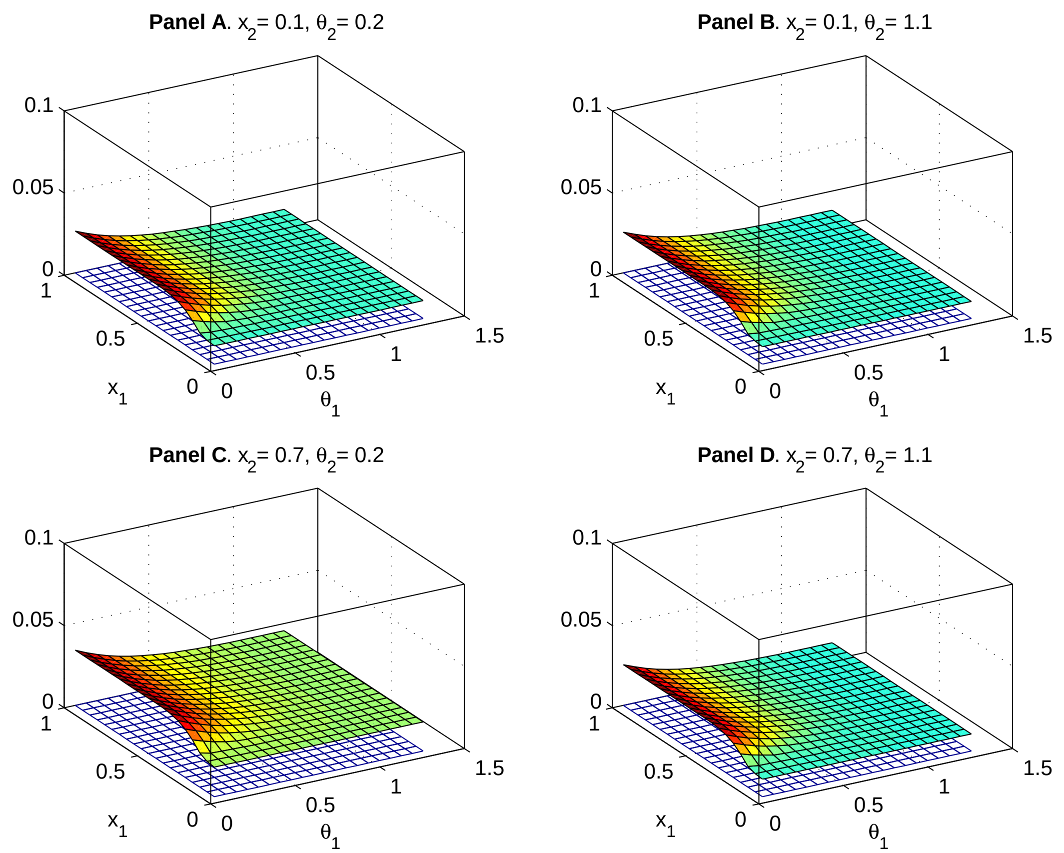



**Figure 5: Value of Tax-Timing Options, $T = 10$. Intermediate consumption Case.**
The value of the tax-timing option is computed as the extra percentage wealth necessary to make the no-tax investor indifferent to holding the taxable portfolio. The figure plots values as a function of the basis and initial holding in asset 1. Every panel plots the option value as a function of holdings in asset 1 ($x_1$) and its relative basis to price ratio ($\theta_1$). Panel A shows the case with low holdings and a capital gain in asset 2. Panel B shows the case with low holdings and a capital loss in asset 2. Panel C shows the case with high holdings and a capital gain in asset 2. Panel D shows the case with high holdings and a capital loss in asset 2.

($\theta_1 \geq 1$). This is due to the possibility of wash sales which, as we commented above, lead to identical portfolio choices from every loss position. The quantity of the tax rebate gained is different for every loss position, but normalizing by the liquidating wealth absorbs this effect and leads to a flat surface in the loss region.

### Sensitivity Analysis

We now perform comparative statics on some parameters of the model in order to assess their impact on the value of the option to defer. In particular we look at the effect of a change in (i) the volatility $\sigma_i$, $i = 1, 2$ of the risky assets, (ii) the coefficient of relative risk aversion $\gamma$, and (iii) the correlation coefficient $\rho$. For all these cases we compare the case of intermediate consumption (retired investor) to the case of no-intermediate consumption (young investor).

In Table 2 we report the value of the tax-deferral option as a function of the volatility in the two risky assets and the coefficient of relative risk aversion $\gamma$. Panel A refers to the case of intermediate consumption and Panel B refers to the case of no-intermediate consumption. The entries in Panel B (no-intermediate consumption) are considerably higher than the entries in panel A (intermediate consumption), reinforcing the fact that forced liquidation to finance consumption substantially reduces the tax-deferral option value.

In both panels of Table 2, the value of the tax deferral option is decreasing in the volatility of the assets and in the risk aversion of the investor. The reason for this can be explained with an analogy to option pricing theory if we interpret the volatility of the assets and the risk aversion as component of the exercise price of the deferral option. The option to defer is in the money when there is an embedded gain and is gradually more and more valuable the higher the embedded gain. If the investor decides to exercise this option (i.e., if he decides to defer the embedded gain) he will have to pay a price in terms of lost diversification. Such a diversification price is higher (i.e. the option is less valuable) the higher is the volatility and the higher is the risk aversion. By exercising the deferral option, the investor is implicitly accepting a larger fraction of an increasingly risky asset. In an intertemporal problem the tax-deferral option is actually a compound option. In this context, the moneyness tomorrow depends on the basis and the asset holding today, as well as the volatility, which affects the basis and asset holding tomorrow. Risk aversion affects the payoff that we get in the future by exercising the option.

In Table 3 we report the sensitivity of the value of the tax-deferral option to changes in the correlation between the risky assets. The second and third columns refer to the case of intermediate consumption and no intermediate consumption respectively. Again notice how the absence of intermediate consumption more than doubles the value of the tax-deferral option. In both cases, the value of the tax-deferral option decreases as the correlation increases. When the underlying assets are highly correlated, holding a portfolio of tax-timing options is less valuable since the two options tend to be in the money or out of the money at the same time. This, in turn, leads to a higher expected diversification cost of exercising the options. In contrast, when assets are negatively correlated, the investor can fully exploit the benefit of optimally and independently exercising the tax-deferral option separately on the underlying assets. Due to the tax-forgiveness provision, it is obvious that the investment horizon has an effect on the value of the tax-deferral option. We confirm that this value is significantly higher for the shorter horizon where the terminal condition has a greater impact. Similarly, ignoring intermediate consumption magnifies the value of the option to defer taxes.[15]

## 4 Value of flexibility in optimal tax realization policies

So far we have assessed the flexibility of exercising the tax-timing option by simply comparing the certainty equivalent across two tax regime: realization vs. accrual. To fully appreciate and value the flexibility an additional asset provides in terms of optimal tax-timing realization, it is important to devise a way to analyze the *incremental benefit* from being able to trade in multiple asset, taking for given the tax regime for capital gains.

Without capital gains taxes, two-fund monetary separation holds (see, for example, Cass and Stiglitz (1970)), i.e., trading in two (or more) securities and a riskless asset is equivalent to trading in the riskless asset and in a fund that combines the two securities in a fixed proportion. In this section we rely on this natural benchmark to assess the effect of flexibility in tax realization polices that an additional asset provides.

The optimal portfolio we solved in the previous section, represents a *first best* solution. If the investor is capable of implementing such strategies or, alternatively, if he hires a professional who commits to such strategies then the optimal portfolio derived above is the best the investor can hope for. When these two assumptions are not satisfied the strategy of the agent (professional money manager) can differ from the first best and the principal (investor) is left to bear the agency cost of such suboptimal strategy.

In the context of our problem, we are interested in analyzing the tax-consequences of suboptimal strategies by portfolio managers. For simplicity we will refer to the situation in which investor delegates the management of his wealth as a *fund* and for the It is clear

[15] Sensitivity analysis on the length of the investment horizon and on the effect of intermediate consumption are available on request.

that by trading in a fund, the investor gives up the *flexibility* provided by the portfolio of tax-deferral options available when trading in the constituent assets. The nature and magnitude of this loss of flexibility is heavily dependent on the tax treatment of capital gains within a fund.

According to the U.S. tax code, to avoid corporate taxation, mutual funds are required to distribute a minimum of 90% of their income and capital gains to shareholders.[16]

An investor who invests in a fund is therefore exposed to potentially sub-optimal tax realization policies of the fund manager. To fully assess the benefit of trading each asset individually we consider two hypothetical experiments. In the first, we compare the optimal strategies with full flexibility and multiple assets to a fund that has a pre-specified (sub-optimal) portfolio target and therefore realizes the gains and losses necessaries to achieve such goal. In the second set of experiment we consider the case of a "fictitious" securities which is obtained by a combination, with fixed weight, of the two existing risky assets. Investors can either trade in the two assets separately or in this composite security. Unlike the previous case, we allow investors to optimally realize taxes on the composite security. However, the fact that the two assets are bundled together is going to affect the benefit from a fully flexible realization policy. The two experiments shed light on two different aspect of the optimal portfolio/realization policies: in the first case, the money manager has flexibility in trading each asset individually but follows a suboptimal realization policy in the second case the money manager (or the investor) optimally exercises his tax-timing options but the fact that assets are tied together in a fixed proportion prevents him from fully benefit from an optimal tax realization policy.

We perform two sets of experiments to assess the tax-cost of delegated investment. In the first set of experiments, we look at the certainty equivalent cost of investing in a fund which follows a pre-specified trading strategy or has asset in fixed proportions. In the second set of experiments we perform a simulation exercise and compare (i) the distribution of terminal wealth and (ii) the total tax paid by investing directly in the two assets vs. investing in a mutual fund following a variety of investment strategies.

[16]It is beyond the scope of this paper to provide a full characterization of the tax treatment within a mutual fund.

We take as reference the following strategies followed by a mutual fund: (i) Fixed proportion of assets with rebalancing (as in Leland (2000)); (ii) Buy-and-hold strategies; (iii) Momentum strategies (buy past winner, sell past losers); (iv) Contrarian strategies.

In both these experiments we assume that the investor does not consume out of his financial wealth.

## 4.1 Wealth Cost of Delegated Investing

Let us consider the portfolio problem described in Section 2 where we add to the menu of financial assets a mutual fund trading in the two existing risky securities. We assume that the objective of the mutual fund manager is to follow one of the strategies pre-specified above. We further assume that all the realized gains and losses generated by trades performed by the manager are distributed to the the investor at the end of every period.[17] In other words, the investor in a mutual fund is forced to realize gains on trades that have been delegated to the manager. In particular, we assume that the portfolio manager in our experiment is offering to maintain the risky assets in the Merton proportions, i.e., the portfolio weights of risky assets that would prevail if we were to solve the problem in a world without tax frictions.

In Figure 6 we investigate the magnitude of the loss of flexibility incurred by investing in a mutual fund compared to investing separately in the two constituent assets. If two-fund monetary separation held, this flexibility option would be worthless. The loss is computed as the certainty equivalent costs incurred by trading in a mutual fund versus trading in the two constituent asset. As in the previous section, the certainty equivalent cost is defined as the amount of additional wealth necessary to make the mutual fund investor as well off as the investor in the two constituent assets. In each panel we report the flexibility loss as a function of holding ($x_1$) and basis-to-price ($\theta_1$) of asset 1 for a fixed holding and basis-to-price ratio in asset 2. Figure 6 confirms the fact that the two-asset case is always preferred to the mutual fund case. The loss of flexibility is larger when the assets have embedded capital gains (North-West corner of each panel). This is intuitive since, in these regions, the tax-deferral options embedded in each security are most valuable, and the

[17] According to the existing tax code, regulated investment companies like open-ended mutual funds cannot distribute net capital losses. By ignoring this in our analysis we are actually biasing downwards the assessment of the flexibility loss induced by mutual fund trading in the presence of capital gains taxes.

mutual fund investor can not take full advantage of them. This also is consistent with the higher magnitude of the certainty equivalent costs observed in Panels A and C, where the second asset has an embedded gain, as opposed to Panels B and D where the second asset has a loss.

Comparing Figures 5, showing the value of the tax-deferral option for the two-asset case with Figure 6, showing the marginal value of trading in the two asset as opposed to trading in a mutual fund, we notice that the magnitude of the certainty equivalent cost in the former is much higher than in the latter. For example, the loss of flexibility by delegating investment to a mutual fund manager when both assets have a gain is less than 3% of current wealth. We conclude from this that, in the current parameterization, most of the value of optimal tax-deferral is already captured by the mutual fund. Only a small additional benefit is attributable uniquely to the flexibility of trading in the two assets independently. It is important to point out however that, since we assume that mutual funds in our experiments are allowed to pass net losses to their shareholders, these values represent a conservative estimate of the flexibility loss due to the presence of capital gains taxes.

Table 4 and 5 report the value of the flexibility option at the Merton's portfolio when the assets do not embed gains or losses. We notice that the value of the flexibility option is decreasing in the volatility and correlation of the asset as well as in the risk aversion of the investor. When assets are positively correlated, the numbers show that the loss in value incurred by an investor who delegates trades to a mutual fund is not dramatic. For an investor with risk aversion $\gamma = 5$, if the volatility of the assets is 25% and the correlation is $\rho = 0.5$, Table 4 show that the loss in value incurred in a ten-period portfolio problem is less than 2% of current wealth. However, if assets are negatively correlated, the loss in flexibility can be as high as 10% of current wealth, as Table 5 illustrates.

To conclude, let us think of the investor solving the portfolio problem in the two constituent assets as a fund manager implementing a tax-efficient portfolio strategy whose objective is the maximization of the shareholder intertemporal utility.[18] In contrast, let us think of the fixed-proportion mutual fund manager as of an agent who trades disregarding the tax consequences of his strategy on the fund shareholders. Then, the certainty equiva-

[18] It would be interesting to investigate whether it is possible to design a contract between the shareholder and the fund-manager which implements such an outcome.

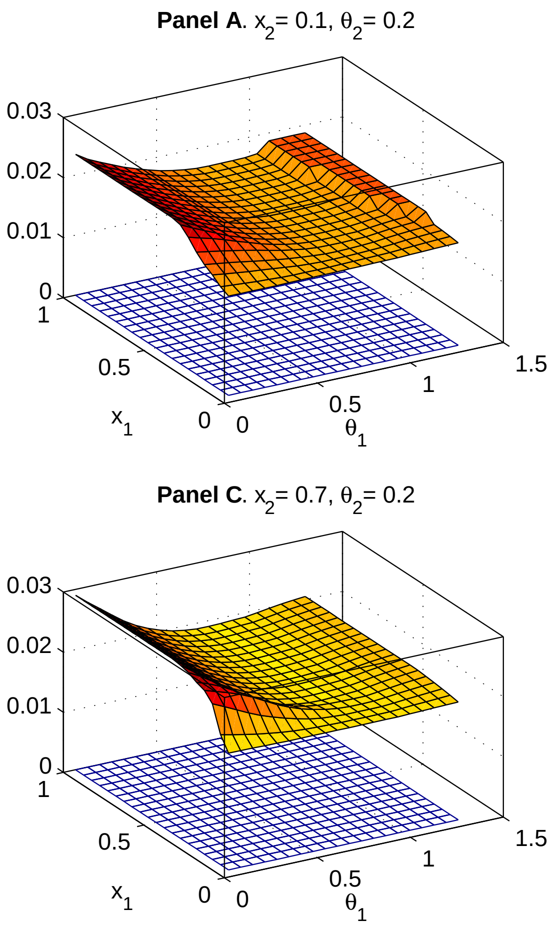



**Figure 6: Value of multi-asset flexibility.**

This Figure shows the certainty equivalent cost of investing in a mutual fund which which passes all gains and losses to the shareholder. Certainty equivalent costs are defined as the percentage additional wealth necessary to make the mutual fund investor indifferent to holding the two taxable assets individually. Positive values indicate states where the two-asset case is preferred. Every panel plots the certainty equivalent costs as a function of holdings in asset 1 ($x_1$) and its relative basis to price ratio ($\theta_1$). Panel A shows the case with low holdings and a capital gain in asset 2. Panel B shows the case with low holdings and a capital loss in asset 2. Panel C shows the case with high holdings and a capital gain in asset 2. Panel D shows the case with high holdings and a capital loss in asset 2. Parameter values are as specified in Table 1.

lent costs analyzed in this section can be easily reinterpreted as the costs incurred by not being able to invest in a tax-efficient mutual fund.

## 5 Conclusions

We solve the portfolio allocation problem in the presence of capital gains taxes for a risk-averse investor who can trade in two risky assets and a riskless security. We interpret the effective capital gains tax burden as an endogenous form of transaction cost on sales and thereby formulate the model as a traditional portfolio problem with state dependent transaction costs. The optimal trading strategies are derived by solving a dynamic programming problem in discrete time with a finite horizon. We solve the portfolio problem for two possible scenarios. In the first scenario, the investor needs to sell financial assets to finance consumption. In the second scenario the investor consumes only at the end of the investment horizon. The former is appropriate for agents in the retirement stage while the latter is a proxy for agents who derive income from non-financial sources (e.g. labor).

In both scenarios, the optimal trading strategy is characterized by a region of no-trades where the investor exercises the option to defer the realization of capital gains. In doing so, the investor trades-off optimal tax-realization with optimal diversification. We show that the diversification cost of capital gain taxes are particularly severe in the case in which investors do not consume out of their financial wealth. To assess the magnitude of the value of the tax-deferral option we compute the certainty-equivalent cost of incurring taxation on accrual rather than taxation on realization. We show that the value of the tax-timing option decreases with risk aversion and as the volatility and the correlation between the assets increases. Not surprisingly, the value of the tax-deferral option is higher in the case of no-intermediate consumption.

To further examine the impact of the tax-timing option, we compare the two-asset case with a stylized forms of open-ended mutual fund that passes dividend, capital gains and losses to the shareholders. This instrument allows us to characterize the value of the flexibility option available to the two-asset investor as well as the cost incurred by investing in a non-tax efficient mutual fund. We find that the loss in flexibility incurred by an investor trading in a non tax-efficient mutual fund is small when assets are highly positively correlated but can be considerable as the correlation among assets decreases.

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# A Appendix: Tables and Figures

**Table 1: Benchmark Case Parameter Values for the Numerical Solution of the Two-Asset Case. The investment horizon is $T$. The parameters $\mu_i$, $d_i$, and $\sigma_i$ give the cum-dividend mean, dividend yield, and standard deviation of asset $i$ returns for $i = 1, 2$. The correlation coefficient is $\rho$. The tax rate on capital gains is $\tau_g$. The risk aversion parameter is $\gamma$.**

| Parameter | Value |
|---|---|
| $T$ | 10 |
| $\mu_1$ | 0.10 |
| $\mu_2$ | 0.10 |
| $\sigma_1$ | 0.30 |
| $\sigma_2$ | 0.30 |
| $\rho$ | 0.50 |
| $\tau_g$ | 0.25 |
| $\gamma$ | 3.00 |

**Table 2:** **Value of Tax-Deferral Option. The table reports the value of the tax-timing option for different levels of the volatility of the two risky assets, different level of risk aversion and different maturity. The value is computed as the extra percentage wealth necessary to make an individual paying taxes on accrual indifferent to holding the tax-on-realization portfolio. For every case, the option value is computed at the point in which the investor holds the Merton's portfolio and the assets do not embed any gain or loss ($\theta_1 = 1, \theta_2 = 1$). Other parameters are as in Table 1.**

| $\sigma_1 = \sigma_2$ | $\gamma = 3.0$ | $\gamma = 5.0$ |
|---|---|---|
| *Panel A. Intermediate Consumption* | | |
| 0.25 | 1.82% | 0.97% |
| 0.30 | 1.26% | 0.68% |
| 0.35 | 0.93% | 0.51% |
| *Panel B. No Intermediate Consumption* | | |
| 0.25 | 7.50% | 4.11% |
| 0.30 | 5.61% | 3.05% |
| 0.35 | 4.37% | 2.39% |

**Table 3:** **Value of Tax-Deferral Option. The table reports the value of the tax-timing option for different levels of correlation between the two risky assets and for different maturities. The value is computed as the extra percentage wealth necessary to make an individual paying taxes on accrual indifferent to holding the tax-on-realization portfolio. Asset holdings are set such that the Merton portfolio is held in each case and the basis to price ratio is $\theta_1 = 1, \theta_2 = 1$. Other parameters are as in Table 1.**

| $\rho$ | Option Value | |
|---|---|---|
| | *Intermediate Consumption* | *No Intermediate Consumption* |
| -0.50 | 6.06% | 12.27% |
| 0.00 | 2.10% | 5.11% |
| 0.50 | 1.26% | 3.05% |

**Table 4:** **Value of Flexibility Option. The table reports, for different values of risk aversion and volatility of the assets, the loss in value incurred by an investor who decides to invest in a mutual fund instead of trading in the two assets separately. These values are computed as the extra percentage wealth necessary to make an individual investing in a mutual fund indifferent to trade in the two assets independently. For every case, the option value is computed at the point in which the initial portfolio $(x_1, x_2)$ corresponds to the Merton's portfolio and the assets do not embed any gain or loss ($\theta_1 = 1, \theta_2 = 1$). Other parameters are as in Table 1.**

| $\sigma_1 = \sigma_2$ | $\gamma = 3.0$ | $\gamma = 5.0$ | $\gamma = 7.0$ |
|---|---|---|---|
| 0.25 | 3.98% | 1.97% | 1.66% |
| 0.30 | 2.67% | 1.33% | 1.06% |
| 0.35 | 1.94% | 0.96% | 0.74% |

**Table 5: Value of Flexibility Option. The table reports, for different levels of correlations between assets, the loss in value incurred by an investor who decides to invest in a mutual fund instead of trading in the two assets separately. The value is computed as the extra percentage wealth necessary to make an individual paying taxes on accrual indifferent to holding the tax-on-realization portfolio. Asset holdings are set such that 20% of wealth is held in each asset (i.e. the Merton portfolio, $x_1 = .2, x_2 = .2$) and the basis to price ratio is $\theta_1 = 1, \theta_2 = 1$. Other parameters are as in Table 1.**

| $\rho$ | Option Value |
|---|---|
| -0.50 | 9.74% |
| 0.00 | 3.12% |
| 0.50 | 1.54% |
| 0.90 | 0.44% |